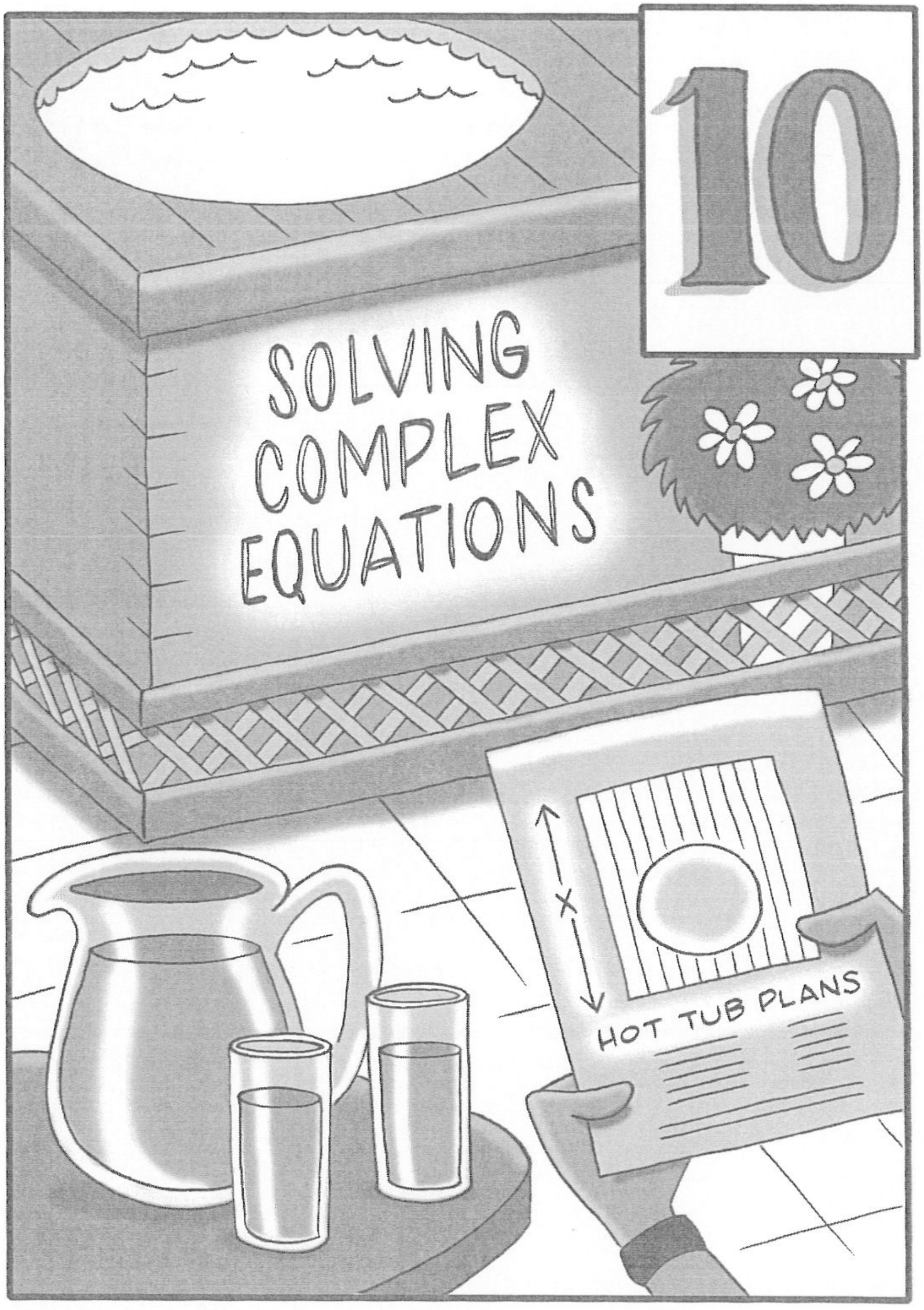
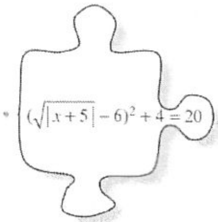
Team #___



10.2.3 How can I solve it?



Multiple Methods for Solving Equations

So far in this course you have developed several different methods for solving equations, such as adding numbers and variables to both sides of the equation or multiplying each term by a number to eliminate fractions. But how would you solve a complicated equation such as the one shown below?

$$(\sqrt{|x+5|}-6)^2+4=20$$

By looking at equations in different ways, you will be able to solve complicated equations quickly and easily. These new approaches will also allow you to solve new kinds of equations you have not studied before. As you solve equations in today's lesson, ask your teammates these questions:

How can you see it?

Is there another way?

10-46. DIFFERENT METHODS TO SOLVE AN EQUATION

In just a few more lessons you will be able to solve the equation $(\sqrt{|x+5|}-6)^2+4=20$. This equation is complex and will require you to look at solving an equation in new ways. To be prepared for other strange and unfamiliar equations, you will first examine all of the solving tools you currently have by solving a comparatively easier equation:

$$4(x+3)=20$$

Your Task: With your team, solve $4(x+3)=20$ for x in *at least* two different ways. Explain how you found x in each case and be prepared to share your explanations with the class.

Further Guidance

10-47. SOLVING BY REWRITING

David wants to find x in the equation $4(x+3)=20$. He said, "*I can rewrite this equation by distributing the 4 on the left-hand side.*" After distributing, what should his new equation be? Solve this equation using David's method.

10-48. SOLVING BY UNDOING

Juan says, "I see the whole thing a different way." Here is how he explains his approach to solving $4(x+3)=20$, which he calls "undoing": "*Instead of distributing first, I want to eliminate the 4 from the left side by undoing the multiplication.*"

a. What can Juan do to both sides of the equation to remove the 4? Why does this work?

b. Solve the equation using Juan's method. Did you get the same result as David?

c. Why is it appropriate for this method to be called "undoing"?

10-49. SOLVING BY LOOKING INSIDE

Kenya said, "I solved David's equation in a much quicker way!" She solved the equation $4(x+3)=20$ with an approach that she calls "looking inside." Here is how she described her thinking: "*I think about everything inside the parentheses as a group. After all, the parentheses group all that stuff together. I think the contents of the parentheses must be 5.*"

a. Why must the expression inside the parentheses equal 5?

b. Write an equation that states that the contents of the parentheses must equal 5. Then solve this equation. Did you get the same result as with David's method?

Further Guidance section ends here.

10-50. THE THREE METHODS

a. Find the Math Notes box for this lesson and read it with your team.

b. Match the names of approaches on the left with the examples on the right.

1. Rewriting
2. Looking inside
3. Undoing

i. "If $3+(4n-4)=12$, then $(4n-4)$ must equal 9..."

ii. "Subtracting is the opposite of adding, so for the equation $3(x-7)+4=23$, I can start by subtracting 4 from both sides..."

iii. "This problem might be easier if I turned $4(2x-3)$ into $8x-12$..."

10-51. For each equation below, decide whether it would be best to rewrite, look inside, or undo. Then solve the equation, showing your work and writing down the name of the approach you used. Check your solutions, if possible.

a. $\frac{2x-8}{10}=6$ b. $4+(x\div 3)=9$ c. $\sqrt{3x+3}=6$

d. $7^{x+3}=7^{3x}$ e. $\sqrt{x}+4=9$ f. $\frac{x}{3}-\frac{x}{9}=6$

10-52. Consider the equation $(x-7)^2=9$.

a. Solve this equation using *all three* approaches studied in this lesson. Make sure each team member solves the equation using all three approaches.

b. Did you get the same solution using all three approaches? If not, why not?

c. Of the three methods, which do you think was the most efficient method for this problem? Why?

METHODS AND MEANINGS

MATH NOTES

Methods to Solve Single-Variable Equations

Here are three different methods you can use to solve single-variable equations:

Rewriting: Use algebraic techniques to rewrite the equation. This will often involve using the Distributive Property to get rid of parentheses. Then solve the equation using solution methods you know.

$$\begin{aligned} 5(x-1) &= 15 \\ 5x-5 &= 15 \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

Looking inside: Choose a part of the equation that includes the variable and is grouped together by parentheses or another symbol. (Make sure it includes *all* occurrences of the variable!) Ask yourself, "What must this part of the equation equal to make the equation true?" Use that information to write and solve a new, simpler equation.

$$\begin{aligned} 5(x-1) &= 15 \\ 5(\ \ 3\ \ ) &= 15 \\ x-1 &= 3 \\ x &= 4 \end{aligned}$$

Undoing: Start by undoing the *last* operation that was done to the variable. This will give you a simpler equation, which you can solve either by undoing again or with some other approach.

$$\begin{aligned} 5(x-1) &= 15 \\ \tfrac{5(x-1)}{5} &= \tfrac{15}{5} \\ x-1 &= 3 \\ +1 &= +1 \\ x &= 4 \end{aligned}$$

Review & Preview

10-53. Solve each equation by first rewriting the expressions in each part with the same base. Refer to problem 10-24 if you need a reminder.

a. $8^x = 2^6$ b. $9^2 = 3^{2x+1}$ c. $4^{2x} = \left(\frac{1}{2}\right)^{x+5}$

10-54. For the equation $\frac{3}{200} + \frac{x}{50} = \frac{7}{100}$:

a. Find a simpler, equivalent equation (i.e., an equivalent equation with no fractions) and solve for x.

b. Which method listed in this lesson's Math Notes box did you use in part (a)?

10-55. Mr. Nguyen has decided to divide \$775 among his three daughters. If the oldest gets twice as much as the youngest, and the middle daughter gets \$35 more than the youngest, how much does each child get? Write an equation and solve it. Be sure to identify your variables.

10-56. How many terms are in the arithmetic sequence shown below?

$$15, 7, -1, -9, \ldots, -225$$

10-57. Marisol and Mimi walked the same distance from their school to a shopping mall. Marisol walked 2 miles per hour, while Mimi left 1 hour later and walked 3 miles per hour. If they reached the mall at the same time, how far from the mall is their school?

10-58. Over a three-year period, the value of a vintage VHS tape of "Charlene's Greatest Hits" decreased from \$20 to \$1.99.

a. What was the annual multiplier and annual percent decrease?

b. Write the equation of the function that describes this situation.

10-59. Delenn is re-examining the difference in backpacks among different grade levels at her school. (She previously collected data in Lesson 8.1.3.) Now she has collected a new random sample of 100 students to see if there are categorical relationships between carrying backpacks and graduating classes.

	Freshmen	**Sophomore**	**Junior**	**Senior**
Backpack	8	16	18	19
No Backpack	3	6	14	16
TOTAL	**11**	**22**	**32**	**35**

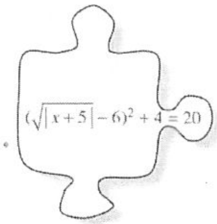
a. Based on her sample, what percentage of students do not carry a backpack at school?

b. If a junior is chosen, what is the probability they are carrying a backpack?

c. If a student is not carrying a backpack, what is the probability they are a junior or senior?

d. Is there a relationship between graduating class and carrying a backpack at school? Show your evidence.

10.2.6 Which method is best?



More Solving and an Application

Recently you investigated three different approaches to solving single-variable equations: rewriting, looking inside, and undoing. Today you will use those approaches to solve new kinds of equations you have not solved before. You will also use your equation-writing skills to write an inequality for an application. As you work today, ask yourself these questions:

How can I represent it?

What is the best approach for this equation?

Have I found all of the solutions?

10-88. Solve these equations. Each time, be sure you have found all possible solutions. Check your work and write down the name of the method(s) you used.

a. $|4x+20|=8$

b. $(x-13)^3=8$

c. $2\sqrt{x-4}=14$

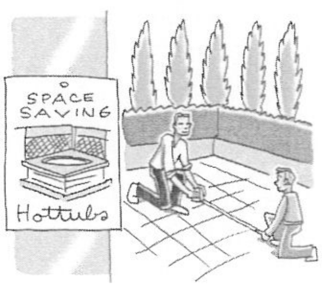
d. $6|x-8|-4=14$

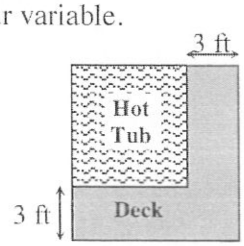
e. $3(x+12)^2=27$

f. $6^{x+9}=36^x$

10-89. RUB A DUB DUB

Ernie is thinking of installing a new hot tub in his backyard. The company he will order it from makes square hot tubs, and the smallest tub he can order is 4 feet by 4 feet. He plans to add a 3-foot-wide deck on two adjacent sides, as shown in the diagram below. If Ernie's backyard (which is also a square) has 169 square feet of space, what are the possible dimensions that his hot tub can be? Write and solve an inequality that represents this situation. Be sure to define your variable.





MATH NOTES

METHODS AND MEANINGS

Solving Absolute-Value Equations

To solve an equation with an absolute value algebraically, first "isolate" the absolute value on one side of the equation.

$$5|2x+3|-6=29$$
$$5|2x+3|=35$$
$$|2x+3|=\tfrac{35}{5}$$
$$|2x+3|=7$$

Determine the possible values of the quantity inside the absolute value.

For example, if $|2x+3|=7$, then the quantity $(2x+3)$ must equal 7 or –7.

With these two values, set up new equations and solve as shown below.

$$|2x+3|=7$$

$2x+3=7$	or	$2x+3=-7$
$2x=4$		$2x=-10$
$x=2$		$x=-5$

Note that distributing over an absolute value is not allowed.
For example,

$$-2|3+1| \neq |-2(3)+-2(1)|$$

Review & Preview

10-90. Sketch a graph of the inequality below. Shade the region containing the solutions of the inequality.

$$y>(x-4)(x+3)$$

10-91. Jessie looked at the rule $y=(x-11)^2+4$ and stated, "The graph of this rule has no x-intercepts!"

a. How could she know without graphing?

b. Find the roots (real or imaginary).

10-92. Solve these equations, if possible. Be sure to find all possible solutions. Check your work and write down the name of the method(s) you used.

a. $9(x-4)^2=81$

b. $|x-6|=2$

c. $5=2+\sqrt{3x}$

d. $2|x+1|=-4$

10-93. Solve the inequalities below. Show your solutions on a number line.

a. $6x-1<11$

b. $\frac{1}{3}x\geq 2$

c. $9(x-2)>18$

d. $5-\frac{x}{4}\leq\frac{1}{2}$

10-94. An exponential function of the form $f(x)=ab^x$ passes through the points $(2,2)$ and $(4,\frac{1}{2})$.

a. Find the equation of the function.

b. Use the equation to sketch a graph of the function.

10-95. Factor each polynomial completely.

a. $6x^2+5x-6$

b. $8x^2-50$

c. $2x^3+2x^2-112x$

d. $9x^2-24x+16$

10-96. Solve each equation. Identify the solutions as rational or irrational numbers.

a. $x(3x-4)=5$

b. $(3x-2)^2=\frac{1}{4}$

Chapter 10 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, a list of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Learning Log Entries

- Lesson 10.2.4 – Number of Solutions
- Lesson 10.2.5 – The Number System
- Lesson 10.3.1 – Intercepts and Intersections
- Lesson 10.3.3 – Solving Inequalities with Absolute Value

Math Notes

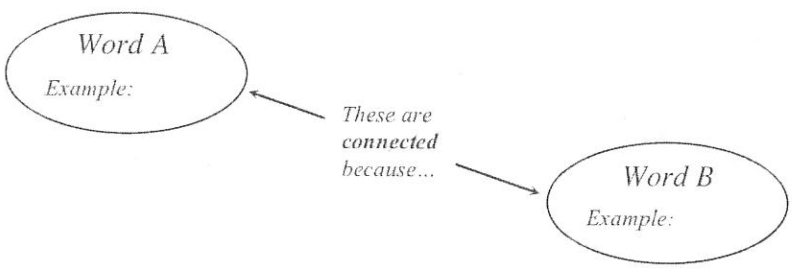
- Lesson 10.2.1 – Equivalent Equations
- Lesson 10.2.2 – Solving Equations with Fractions (also known as Fraction Busters)
- Lesson 10.2.3 – Methods to Solve Single-Variable Equations
- Lesson 10.2.4 – Forms of a Quadratic Equation
- Lesson 10.2.5 – The Number System
- Lesson 10.2.6 – Solving Absolute Value Equations

② MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

absolute value	factored form	boundary point
equivalent equations	exponent	number system
Fraction Busters	inequality	association
number line	looking inside	relative frequency
standard form for quadratics	real number	quadratic equation
Quadratic Formula	rewriting	rational number
base	undoing	perfect square form
categorical data	intersection	two-way table
irrational number	intercept	imaginary number
simplifying	solution	integer

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the model below. A word can be connected to any other word as long as you can justify the connection. For each key word or idea, provide an example or sketch that shows the idea.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

Show your solution to problem 10-135 and explain it in detail to showcase your growth in solving equations that began in Chapter 3.

④ WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 10-143. Solve the equations below using any method. How many solutions does each problem have?

a. $\frac{x}{2}+\frac{x}{3}=2$

b. $\sqrt{x-5}+10=15$

c. $|x-7|=22$

d. $(3x+7)^2=144$

CL 10-144. Lately there have been a number of times when the sound quality of the news interviews on the school's video station has been unfit to broadcast. One source of the sound problem might be that one or two of the microphones is not working well. Brendan collected the following data from the last broadcast season:

Interview Sound was:	Mic A	Mic B	Mic C
Good	27	49	33
Unacceptable	6	10	7
TOTAL	**33**	**59**	**40**

a. What is the probability that a randomly selected show will have an unacceptable sound quality?

b. Is there an association between the sound quality and the microphone used? Should Brendan keep searching for the source of the sound problem or has he found it?

CL 10-145. For the exponential function $y = 20(1.06)^x$:

a. What are the starting point (y-intercept) and multiplier? Sketch a graph.

b. This function describes the yearly growth of a bank account. What percent interest does the account earn per year?

CL 10-146. Factor each polynomial.

a. $x^2 - x - 56$ b. $3x^2 - 4x + 1$

c. $2x^3 + x^2 + x$ d. $2x^2 - 50$

CL 10-147. Solve each quadratic equation using any method. Describe the answers as rational or irrational numbers.

a. $2x^2 - x - 5 = 0$ b. $4x^2 = 4x - 1$

CL 10-148. For an exponential function of the form $f(x)=ab^x$, $f(3)=24$ and $f(4)=48$. Find an equation that matches this data.

CL 10-149. Use a graph to estimate the solutions to $2^x=x+3$.

CL 10-150. Write and solve an equation to answer the question below.

Lorena and Glenda both work in the library. Lorena has already returned 38 books to the shelves and can shelve 16 books an hour on average. Glenda, on the other hand, has already returned 52 books to the shelves and can shelve 20 books per hour on average. If together they started with a total of 252 books to return on the shelves, how much longer should it take them?

CL 10-151. For the quadratic function $f(x)=(x-3)^2+4$:

a. Identify the vertex and tell if it the maximum or minimum point of the function.

b. Why does $(x-3)^2+4=0$ have no real solutions?

c. Find the imaginary roots of the function.

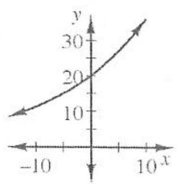
CL 10-152. Write an inequality to represent this situation.

If apples cost \$0.75 each and oranges cost \$0.50 each, what combinations of the fruits can be bought for under \$10?

CL 10-153. Check your answers to each problem above using the table at the end of the closure section. Which problems did you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

Answers and Support for Closure Activity #4
What Have I Learned?

Note: MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice
CL 10-143.	a. $x = \frac{12}{5}$ b. $x = 30$ c. $x = 29$ or -15 d. $x = \frac{5}{3}$ or $-\frac{19}{3}$	Lessons 10.2.2, 10.2.3, and 10.2.4 MN: 10.2.2 and 10.2.3 LL: 10.2.4	Problems 10-39, 10-54, 10-67, 10-83, 10-92, and 10-141
CL 10-144.	a. $\frac{6+10+7}{33+59+40} = \frac{23}{132} = 17\%$ b. See relative frequency table below. The percentage of unacceptable audio was about the same for all three microphones. Any differences are probably due to natural sample-to-sample variability. No association. There is likely some other source for the sound quality problem.	Lesson 10.1.1	Problems 10-29, 10-59, and 10-113
CL 10-145.	a. See graph at right. b. Anything with initial value of 20 and increasing by 6%. 	Chapter 7 Checkpoint 10A LL: 7.1.2 and 7.1.6	Problems CL 7-120, 10-19, 10-58, 10-95, and 10-119
CL 10-146.	a. $(x+8)(x-7)$ b. $(3x-1)(x-1)$ c. $x(2x^2+x+1)$ d. $2(x-5)(x+5)$	Section 8.1 Checkpoint 10B MN: 8.1.4 LL: 8.1.3 and 8.1.5	Problems CL 8-112, CL 9-122, 10-21, 10-69, 10-95, and 10-142

Relative frequency table for CL 10-144 b:

Interview Sound was:	Mic A	Mic B	Mic C
Good	82%	83%	83%
Unacceptable	18%	17%	18%

Problem	Solution	Need Help?	More Practice
CL 10-147.	a. $x=\frac{1\pm\sqrt{41}}{4}$, irrational b. $x=\frac{1}{2}$, rational	Lesson 10.2.5 MN: 10.2.5 LL: 10.2.5	Problems 10-82, 10-96, and 10-115
CL 10-148.	$f(x)=3(2)^x$	Lessons 7.2.1 and 7.2.2 MN: 9.3.1	Problems CL 7-112, CL 9-131, 10-33, 10-44, and 10-94
CL 10-149.	$x\approx 2.5$ or ≈ -2.8	Lesson 10.3.1 LL: 10.3.1	Problems 10-114, 10-130, and 10-140
CL 10-150.	If t = time, $38+16t+52+20t=252$ $t=4.5$ hours	Sections 2.1 and 2.2	Problems CL 3-113, CL 4-119, CL 8-118, CL 9-130, 10-32, 10-42, 10-55, 10-57, and 10-70
CL 10-151.	a. The vertex $(3,4)$ is a minimum. b. The vertex is a minimum and above the x-axis, therefore there are no x-intercepts. c. $3\pm\sqrt{-4}$	Section 8.2 MN: 8.2.4	Problems CL 9-126, 10-84, 10-91, and 10-139(d)
CL 10-152.	If a = # of apples and b = # of oranges: $0.75a+0.50b<10$	Section 9.2	Problems CL 9-128, 10-73, 10-85, and 10-116